IF $a_n = \frac{1}{(2n)!}$, $\frac{a_{n+1}}{a_n} = \frac{\frac{1}{(2(n+1))!}}{\frac{1}{(2n)!}} = \frac{1}{(2(n+1))!} \cdot \frac{(2n)!}{1}$

$$= \frac{(2n)!}{(2n+2)!}$$

$$= \frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}}$$

$$\underset{-1}{\curvearrowright} \quad \underset{-1}{\curvearrowright}$$

$$= \frac{1}{(2n+2)(2n+1)}$$

$$(2n)! \neq 2(n!)$$

eg. $n=2$

$2n=4$

$(2n)! = 4!$

$= 4 \cdot 3 \cdot 2 \cdot 1$

$= 24$

$2(2!)$

$= 2(2 \cdot 1)$

$= 4$

11.8 POWER SERIES ("INFINITE POLYNOMIAL")

DEGREE OF ALL TERMS MUST BE A NON-NEGATIVE INTEGER

"POWER SERIES" (ABOUT 0)

$$\sum_{n=0}^{\infty} c_n x^n$$

$c_n \in \mathbb{R}$ FOR ALL $n \in \mathbb{Z}^{\text{non-neg}}$ OR $\mathbb{Z}^*$

OR $$\sum_{n=0}^{\infty} c_n (x-a)^n \qquad a \in \mathbb{R}$$

"POWER SERIES ABOUT a"

NOTE: IF $x = a$

$$\sum_{n=0}^{\infty} c_n 0^n = c_0 \underbrace{0^0}_{\text{UNDEFINED}} + c_1 0^1 + c_2 0^2 + \cdots = c_0$$

$$\boxed{\text{FOR POWER SERIES, DEFINE } 0^0 = 1}$$

$$1+\frac{2}{x}+\frac{3}{x^2}+\frac{4}{x^3}+\cdots$$

NOT A POWER SERIES

~~DEGREE~~ POWERS OF X IN STANDARD FORM ARE NEGATIVE

$$=1+2x^{-1}+3x^{-2}+4x^{-3}$$

$$1(x-0)+2(x-1)^2+3(x-2)^3+4(x-3)^4+\cdots$$

NOT A POWER SERIES

SUBTRACTING DIFFERENT CONSTANTS FROM EACH X

$$(x+x^2)+(x^2+x^3)+(x^3+x^4)+\cdots \leftarrow \text{NOT A POWER SERIES}$$

$$= x+2x^2+2x^3+2x^4+\cdots \leftarrow \text{POWER SERIES}$$

$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

HOW DO WE FIND c_n?

HOW DO WE CHOOSE a?

FOR WHAT VALUES OF X DOES THE SERIES CONVERGE

FOR WHAT VALUES OF X DOES $\sum_{n=1}^{\infty} \boxed{\frac{(x-3)^n}{n(-2)^{n+1}}}$ CONVERGE ?

a_n

RATIO TEST

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1 \longrightarrow \sum_{n=1}^{\infty} a_n \text{ CONV}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(x-3)^{n+1}}{(n+1)(-2)^{n+1+1}}}{\frac{(x-3)^n}{n(-2)^{n+1}}}\right| = \boxed{\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(n+1)(-2)^{n+2}}\cdot\frac{n(-2)^{n+1}}{(x-3)^n}\right|}$$

$$= \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(x-3)^n}\cdot\frac{n}{n+1}\cdot\frac{(-2)^{n+1}}{(-2)^{n+2}}\right|$$

$|x| < a \Leftrightarrow$
$-a < x < a$

$$= \boxed{\lim_{n\to\infty}\left|(x-3)\cdot\frac{1}{1+\frac{1}{n}}\cdot\frac{1}{-2}\right|}$$

$$= |x-3|\cdot\frac{1}{1+0}\cdot\frac{1}{2}$$

$$= \boxed{\tfrac{1}{2}|x-3|}\boxed{< 1}$$

$\frac{1}{2}|x-3| < 1$

$|x-3| < 2$

$\boxed{-2 < x-3 < 2}$

$\boxed{1 < x < 5}$

FOR $x \in (1,5)$ SERIES CONV (RATIO TEST)

NOTE: IF $\frac{1}{2}|x-3| > 1$, SERIES DIV

IE. $x < 1$ OR $x > 5$ IE. $x \in (-\infty, 1)$ OR $x \in (5, \infty)$

FOR $\boxed{x=1}$ (RATIO TEST INCONCLUSIVE)

$$\sum_{n=1}^{\infty} \frac{(1-3)^n}{n(-2)^{n+1}}$$

$$= \sum_{n=1}^{\infty} \frac{(-2)^n}{n(-2)^{n+1}}$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{1}{-2n}} = \boxed{-\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n} \text{ DIV (p-SERIES)}}$$

$$\boxed{p = 1 \leq 1}$$

FOR $\boxed{x=5}$

$$(ab)^m = a^m b^m$$

$$(-2)^n = ((-1)\cdot 2)^n = (-1)^n 2^n$$

$$\sum_{n=1}^{\infty} \frac{(5-3)^n}{n(-2)^{n+1}}$$

$$= \sum_{n=1}^{\infty} \frac{2^n}{n(-1)^{n+1} 2^{n+1}}$$

$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n}} \leftarrow \text{ALTERNATING}$$

$$\boxed{\begin{array}{l} \frac{1}{2n} > 0 \text{ ON } [1,\infty) \checkmark \\ \text{DECR ON } [1,\infty) \checkmark \\ \to 0 \text{ AS } n \to \infty \checkmark \end{array}}$$

$$\boxed{\text{CONV (ALT)}}$$

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n(-2)^{n+1}}$$

CONV $1 < x < 5$ ~~AND~~ OR $x = 5$

IE. $1 < x \leq 5$

IE $x \in (1, 5]$

DIV $x < 1$ OR $x > 5$ OR $x = 1$

IE. $x \leq 1$ OR $x > 5$

IE. $x \in (-\infty, 1]$ OR $x \in (5, \infty)$